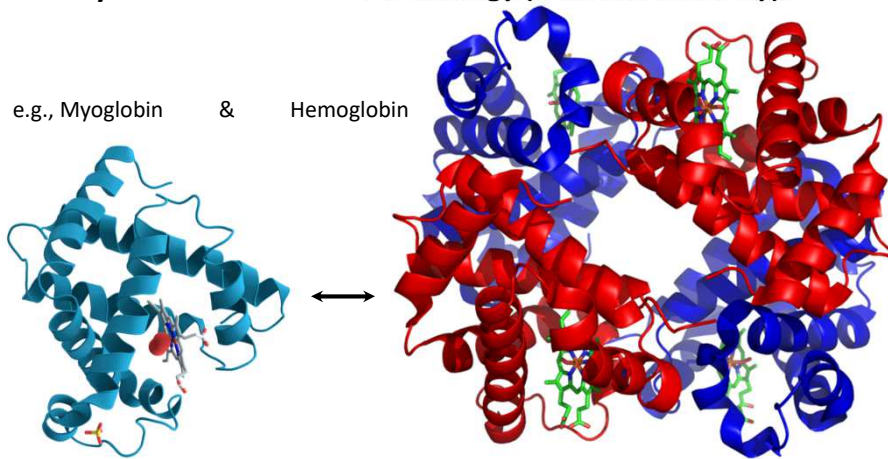


Typically, to be “biologically related” means to share a common ancestor. In biology, we call this *homologous*.

Two proteins sharing a common ancestor are said to be *homologs*.

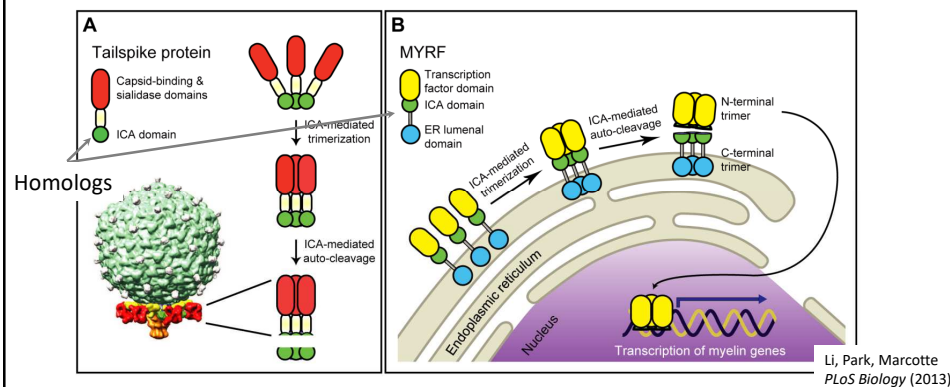
Homology often implies structural similarity & sometimes (not always) sequence similarity. A statistically significant sequence or structural similarity can be used to infer homology (common ancestry).



1

In practice, searching for sequence or structural similarity is one of the most powerful computational approaches to discover a gene’s function. We can often gain insight about a protein from its homologs.

For example, my lab discovered that myelinating the neurons in your brain reuses the same biochemical mechanism that phage use to make capsids. The key breakthrough was recognizing that the human and phage proteins contained homologous domains.



2

**Sequence alignment algorithms such as BLAST, PSI-BLAST, FASTA, and the Needleman–Wunsch & Smith-Waterman algorithms arguably comprise some of the most important driver technologies of modern biology and underlie the sequencing revolution.**

**So, let's start learning bioinformatics algorithms by learning how to align two protein sequences.**

3

**Live demo:**

[http://blast.ncbi.nlm.nih.gov/Blast.cgi?PROGRAM=blastp&BLAST\\_PROGRAMS=blastp  
&PAGE\\_TYPE=BlastSearch&SHOW\\_DEFAULTS=on&LINK\\_LOC=blasthome](http://blast.ncbi.nlm.nih.gov/Blast.cgi?PROGRAM=blastp&BLAST_PROGRAMS=blastp&PAGE_TYPE=BlastSearch&SHOW_DEFAULTS=on&LINK_LOC=blasthome)

MVLSPADKTNVKAAWGKVGAGHAGEYGAEALERMFSLFPTTKTYFPHFDLSHGSAQVKGHG  
KKVADALTNAVAHVDDMPNALSALSDLHAHKLRVDPVNFKLLSHCLLVTLAAHLPAEFTP  
AVHASLDKFLASVSTVLTSKYR

Title: All non-redundant GenBank CDS translations+PDB+SwissProt+PIR+PRF excluding  
environmental samples from WGS projects

Molecule Type: Protein

Update date: 2019/03/22

Number of sequences: 251,406,437

4



Descriptions

Graphic Summary

Alignments

Taxonomy

Alignment view

Pairwise

Download

100 sequences selected

Download

GenPept

Graphics

Next

Previous

Descriptions

hemoglobin alpha 2, partial [synthetic construct]

Sequence ID: [AAX29522.1](#) Length: 143 Number of Matches: 1

Range 1: 1 to 142

GenPept

Graphics

Next Match

Previous Match

Score	Expect	Method	Identities	Positives	Gaps
287 bits(734)	7e-98	Compositional matrix adjust.	142/142(100%)	142/142(100%)	0/142(0%)
Query 1	MVLSPADKTNVKAAWGKVGAGHAGEYGAEALERMF	1	MVLSPADKTNVKAAWGKVGAGHAGEYGAEALERMF	1	
Sbjct 1	MVLSPADKTNVKAAWGKVGAGHAGEYGAEALERMF		MVLSPADKTNVKAAWGKVGAGHAGEYGAEALERMF	1	
Query 61	KKVADALTNVAHVDDMPNALSALSDLHAHKL	61	KKVADALTNVAHVDDMPNALSALSDLHAHKL	61	
Sbjct 61	KKVADALTNVAHVDDMPNALSALSDLHAHKL		KKVADALTNVAHVDDMPNALSALSDLHAHKL	61	
Query 121	AVHASLDKFLASVSTVLT	121	AVHASLDKFLASVSTVLT	121	
Sbjct 121	AVHASLDKFLASVSTVLT		AVHASLDKFLASVSTVLT	121	

7

Protein sequence alignment

Two biologically related proteins with similar sequences:

FlgA1

EAGNVKLKRGRDLTLPRTVLDINQLVDAISLRDLS

PDQPIQLTQFRQAWRVKAGQQRVNVIASGD

++K+K+GRDLTLPP +L+ N A+SLR ++ QP+ R+ W +KAGQ V V+A G+

FlgA2

TLQDIKMKQGRDLTLPFGALLEPNFAQQGAVSLRQINAGQPLTRNMLRRLWIIKAGQDVQVLALGE

Also biologically related (& fold up into the same 3D protein structure):

FlgA1

EAGNVKLKRGRDLTLPRTVLDINQLVDAISLRDLS

PDQPIQLTQFRQAWRVKAGQQRVNVIASGD

A + P +L I+ R L P + I R+AW V+ G V V

FlgA3

LAALKQVTTLIAGKHKPDAMATHABEELQGKI

AKRTLLPGRYIPTAAIREAWLVEQGA

AVQVFFIAG

But these are biologically unrelated (& fold up into unrelated structures):

FlgA1

AGNVKLKRGRDLTLPRTVLDINQLVDAISLRDLS

PDQPIQLTQFRQA-WRVKAGQQRVNVIASGD

AG+V K G + + PRT ++ I+ P PI +++A WRV A + V V+ GD

HvcPP

AGHV--KNGTMRI

VGPRTC

SNVWNGTFPINATTG

PSIPI

PAPNYK

KALWRV

SATEY

VEVVRVGD

(FYI, we'll draw examples from Durbin *et al.*, *Biological Sequence Analysis*, Ch. 1 & 2).

8

**To align two sequences, we need to perform 3 steps:**

- 1. We need some way to decide which alignments are better than others.  
For this, we'll invent a way to give the alignments a "score" indicating their quality.**
- 2. Align the two proteins so that they get the best possible score.**
- 3. Decide if the score is "good enough" for us to believe the alignment is biologically significant.**

9

**To align two sequences, we need to perform 3 steps:**

- 1. We need some way to decide which alignments are better than others.  
For this, we'll invent a way to give the alignments a "score" indicating their quality.**
- 2. Align the two proteins so that they get the best possible score.**
- 3. Decide if the score is "good enough" for us to believe the alignment is biologically significant.**

10

We'll treat mutations as independent events.

This allows us to create an ***additive scoring scheme***.

The score for a sequence alignment will be the sum of the scores for aligning each of the individual positions in two sequences.

**What kind of mutations should we expect?**

11

***Substitutions, insertions and deletions.***

Insertions and deletions can be treated as equivalent events by considering one or the other sequence as the reference, and are usually called ***gaps***.

Diagram illustrating sequence alignment with substitutions and gaps:

```
AGNVKLKRG
AG+V    K G
AGHV - -KNG
```

Arrows point from the labels ***substitution*** and ***gap*** to the corresponding positions in the alignment:

- substitution*** points to the alignment of 'V' and 'V' in the second and third sequences.
- gap*** points to the alignment of '-' and '-' in the second and third sequences.

12

Let's consider two models:

First, a **random** model, where amino acids in the sequences occur independently at some given frequencies.

The probability of observing an alignment between  $x$  and  $y$  is just the product of the frequencies ( $q$ ) with which we find each amino acid.

We can write this as:

$$P(x, y | R) = \prod_i q_{x_i} \prod_j q_{y_j}$$

What's this mean? (pointing to  $P(x, y | R)$ )

What does the capital pi mean? (pointing to  $\prod$ )

What's this mean? (pointing to  $q_{x_i}$ )

What's this mean? (pointing to  $q_{y_j}$ )

13

Second, a **match** model, where amino acids at a given position in the alignment arise from some common ancestor with a probability given by the joint probability  $p_{ab}$ .

So, under this model, the probability of the alignment is the product of the probabilities of seeing the individual amino acids aligned.

We can write that as:

$$P(x, y | M) = \prod_i p_{x_i, y_i}$$

What's this mean? (pointing to  $P(x, y | M)$ )

What does the capital pi mean again? (pointing to  $\prod$ )

What's this mean? (pointing to  $p_{x_i, y_i}$ )

14

To decide which model better describes an alignment, we'll take the ratio:

$$\frac{P(x, y | M)}{P(x, y | R)} = \frac{\prod_i P_{x_i y_i}}{\prod_i q_{x_i} \prod_j q_{y_j}} = \prod_i \frac{P_{x_i y_i}}{q_{x_i} q_{y_i}}$$

What did these mean again?

Such a ratio of probabilities under 2 different models is called an **odds ratio**.

Where else have you heard odds ratios used?

Basically: if the ratio > 1, model *M* is more probable  
if < 1, model *R* is more probable.

15

Now, to convert this to an additive score *S*, we can simply take the logarithm of the odds ratio (called the **log odds ratio**):

$$S = \sum_i s(x_i, y_i)$$

This is just the score for aligning one amino acid with another amino acid:

$$s(a, b) = \log \left( \frac{p_{ab}}{p_a p_b} \right)$$

Here written *a* and *b* rather than *x<sub>i</sub>* and *y<sub>i</sub>* to emphasize that this score reflects the inherent preference of the two amino acids (*a* and *b*) to be aligned.

**Almost done with step 1...**

16



The last trick:

Take a big set of pre-aligned protein sequence alignments (that are correct!) and measure all of the pairwise amino acid substitution scores (the  $s(a,b)$ 's). Put them in a 20x20 **amino acid substitution matrix** :

	A	R	N	D	C	Q	E	G	H	I	L	K	M	F	P	S	T	W	Y	V
A	5	-2	-1	-2	-1	-1	-1	0	-2	-1	-2	-1	-1	-3	-1	1	0	-3	-2	0
R	-2	7	-1	-2	-4	1	0	-3	0	-4	-3	3	-2	-3	-3	-1	-1	-3	-1	-3
N	-1	-1	7	2	-2	0	0	0	1	-3	-4	0	-2	-4	-2	1	0	-4	-2	-3
D	-2	-2	2	8	-4	0	2	-1	-1	-4	-4	-1	-4	-5	-1	0	-1	-5	-3	-4
C	-1	-4	-2	-4	13	-3	-3	-3	-3	-2	-2	-3	-2	-2	-4	-1	-1	-5	-3	-1
Q	-1	1	0	0	-3	7	2	-2	1	-3	-2	2	0	-4	-1	0	-1	-1	-1	-3
E	-1	0	0	2	-3	2	6	-3	0	-4	-3	1	-2	-3	-1	-1	-1	-3	-2	-3
G	0	-3	0	-1	-3	-2	-3	8	-2	-4	-4	-2	-3	-4	-2	0	-2	-3	-3	-4
H	-2	0	1	-1	-3	1	0	-2	10	-4	-3	0	-1	-1	-2	-1	-2	-3	2	-4
I	-1	-4	-3	-4	-2	-3	-4	-4	-4	5	2	-3	2	0	-3	-3	-1	-3	-1	4
L	-2	-3	-4	-4	-2	-2	-3	-4	-3	2	5	-3	3	1	-4	-3	-1	-2	-1	1
K	-1	3	0	-1	-3	2	1	-2	0	-3	-3	6	-2	-4	-1	0	-1	-3	-2	-3
M	-1	-2	-2	-4	-2	0	-2	-3	-1	2	3	-2	7	0	-3	-2	-1	-1	0	1
F	-3	-3	-4	-5	-2	-4	-3	-4	-1	0	1	-4	0	8	-4	-3	-2	1	4	-1
P	-1	-3	-2	-1	-4	-1	-1	-2	-2	-3	-4	-1	-3	-4	10	-1	-1	-4	-3	-3
S	1	-1	1	0	-1	0	-1	0	-1	-3	-3	0	-2	-3	-1	5	2	-4	-2	-2
T	0	-1	0	-1	-1	-1	-1	-2	-2	-1	-1	-1	-1	-2	-1	2	5	-3	-2	0
W	-3	-3	-4	-5	-5	-1	-3	-3	-3	-3	-2	-3	-1	1	-4	-4	-3	15	2	-3
Y	-2	-1	-2	-3	-3	-1	-2	-3	2	-1	-1	-2	0	4	-3	-2	-2	2	8	-1
V	0	-3	-3	-4	-1	-3	-3	-4	-4	4	1	-3	1	-1	-3	-2	0	-3	-1	5

17

This is the **BLOSUM50** matrix.

(The numbers are scaled & rounded off to the nearest integer):

What's the score for aspartate (D) aligning with itself?

How about aspartate with phenylalanine (F)? Why?

	A	R	N	D	C	Q	E	G	H	I	L	K	M	F	P	S	T	W	Y	V
A	5	-2	-1	-2	-1	-1	-1	0	-2	-1	-2	-1	-1	-3	-1	1	0	-3	-2	0
R	-2	7	-1	-2	-4	1	0	-3	0	-4	-3	3	-2	-3	-3	-1	-1	-3	-1	-3
N	-1	-1	7	2	-2	0	0	0	1	-3	-4	0	-2	-4	-2	1	0	-4	-2	-3
D	-2	-2	2	8	-4	0	2	-1	-1	-4	-4	-1	-4	-5	-1	0	-1	-5	-3	-4
C	-1	-4	-2	-4	13	-3	-3	-3	-3	-2	-2	-3	-2	-2	-4	-1	-1	-5	-3	-1
Q	-1	1	0	0	-3	7	2	-2	1	-3	-2	2	0	-4	-1	0	-1	-1	-1	-3
E	-1	0	0	2	-3	2	6	-3	0	-4	-3	1	-2	-3	-1	-1	-1	-3	-2	-3
G	0	-3	0	-1	-3	-2	-3	8	-2	-4	-4	-2	-3	-4	-2	0	-2	-3	-3	-4
H	-2	0	1	-1	-3	1	0	-2	10	-4	-3	0	-1	-1	-2	-1	-2	-3	2	-4
I	-1	-4	-3	-4	-2	-3	-4	-4	-4	5	2	-3	2	0	-3	-3	-1	-3	-1	4
L	-2	-3	-4	-4	-2	-2	-3	-4	-3	2	5	-3	3	1	-4	-3	-1	-2	-1	1
K	-1	3	0	-1	-3	2	1	-2	0	-3	-3	6	-2	-4	-1	0	-1	-3	-2	-3
M	-1	-2	-2	-4	-2	0	-2	-3	-1	2	3	-2	7	0	-3	-2	-1	-1	0	1
F	-3	-3	-4	-5	-2	-4	-3	-4	-1	0	1	-4	0	8	-4	-3	-2	1	4	-1
P	-1	-3	-2	-1	-4	-1	-1	-2	-2	-3	-4	-1	-3	-4	10	-1	-1	-4	-3	-3
S	1	-1	1	0	-1	0	-1	0	-1	-3	-3	0	-2	-3	-1	5	2	-4	-2	-2
T	0	-1	0	-1	-1	-1	-1	-2	-2	-1	-1	-1	-1	-2	-1	2	5	-3	-2	0
W	-3	-3	-4	-5	-5	-1	-3	-3	-3	-3	-2	-3	-1	1	-4	-4	-3	15	2	-3
Y	-2	-1	-2	-3	-3	-1	-2	-3	2	-1	-1	-2	0	4	-3	-2	-2	2	8	-1
V	0	-3	-3	-4	-1	-3	-3	-4	-4	4	1	-3	1	-1	-3	-2	0	-3	-1	5

18

Using this matrix, we can score any alignment as the sum of scores of individual pairs of amino acids.

For example, the top alignment in our earlier example:

```
FlgA1  EAGNVKLKRGRDLTLPPRTVLVDINQLVDAISLRDLSPDQPIQLTQFRQAWRVKAGQRVNVIASGD
      ++K+K+GRDLTLPP  +L+ N   A+SLR ++  QP+      R+ W +KAGQ V V+A G+
FlgA2  TLQDIKMKQGRDLTLPPGALLEPNFAQGAVSLRQINAGQPLTRNMLRRLWIIKAGQDVQVLALGE
```



gets the score:

$$S(\text{FlgA1}, \text{FlgA2}) = -1 - 2 - 2 + 2 + 4 + 6 + \dots = 186$$

We also need to penalize gaps. For now, let's just use a constant penalty  $d$  for each amino acid gap in an alignment, *i. e.*:

the penalty for a gap of length  $g = -g*d$

19

PAM	vs.	BLOSUM
		
<p>Margaret Dayhoff (1925-1983) Developed point accepted mutation matrices (PAM matrices)</p>		<p>Steve and Jorja Henikoff Developed BLOSUM matrices</p>
<p><u>Calibrated for different evolutionary times</u> PAM-<math>n</math> = <math>n</math> substitutions per 100 residues e.g. matrices from PAM1 to PAM250 measure PAM1, calculate higher PAMs from that</p>		<p><u>Calibrated for different % identity sequences</u> BLOSUM-<math>n</math> = for sequences of about <math>n</math> % identity averages substitution probabilities over sequence clusters, gives better estimates for highly divergent cases</p>
<p><u>Explicit model of evolution</u> (calculated using a phylogenetic tree)</p>		<p><u>Implicit model of evolution</u> (calculated from blocks of aligned sequences)</p>

20

**To align two sequences, we need to perform 3 steps:**

1. We need some way to decide which alignments are better than others.  
For this, we'll invent a way to give the alignments a "score" indicating their quality.
2. **Align the two proteins so that they get the best possible score.**
3. Decide if the score is "good enough" for us to believe the alignment is biologically significant.

21

We'll use something called *dynamic programming*.

This is **mathematically guaranteed** to find the best scoring alignment, and uses **recursion**. This means problems are broken into sub-problems, which are in turn broken into sub-problems, etc, until the simplest sub-problems can be solved.

We're going to find the best **local** alignment—the best matching internal alignment—without forcing all of the amino acids to align (i.e. to match **globally**).

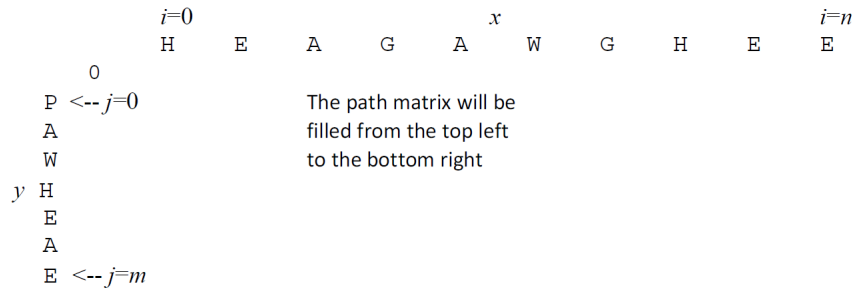
i.e., this  $\longrightarrow$   $\begin{matrix} \text{ATGCAT} \\ \text{ATGCAT} \end{matrix}$

Not this  $\longrightarrow$  ACGTTATGCATGACGTA  
-C---ATGCAT-----T-

22

We'll make a ***path matrix***, showing the possible alignments and their scores. There are simple rules for how to fill in the matrix.

*This will test all possible alignments & give us the top-scoring alignment between the two sequences.*





23

**Here are the rules:**

For a given square in the matrix  $F(i,j)$ , we look at the squares to its left  $F(i-1,j)$ , top  $F(i,j-1)$ , and top-left  $F(i-1,j-1)$ . Each should have a score.

We consider 3 possible events & choose the one scoring the highest:

- |                               |   |  |   |
|-------------------------------|---|--|---|
|                               |  |  |  |
| (1) $x_i$ is aligned to $y_j$ |   |  | $F(i-1, j-1) + s(x_i, y_j)$   |
| (2) $x_i$ is aligned to a gap |   |  | $F(i-1, j) - d$   |
| (3) $y_j$ is aligned to a gap |   |  | $F(i, j-1) - d$   |

For this example, we'll use  $d = 8$ . We also set the left-most & top-most entries to zero.

24

**Just two more rules:**

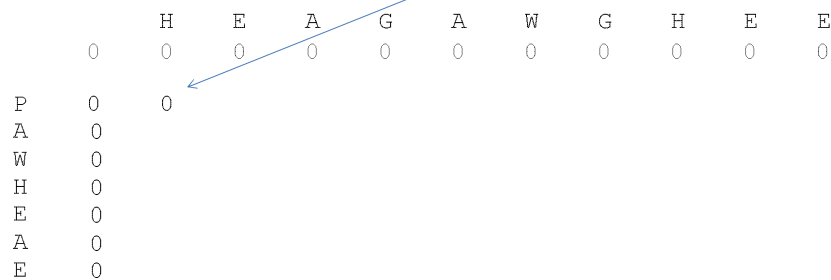
**If the score is negative, set it equal to zero.**

At each step, we also keep track of which event was chosen by **drawing an arrow from the cell we just filled back to the cell which contributed its score to this one.**

**That's it! Just repeat this to fill the entire matrix.**

25

Here we go! Start with the borders & the first entry.



		H	E	A	G	A	W	G	H	E	E
	0	0	0	0	0	0	0	0	0	0	0
P	0	0									
A	0										
W	0										
H	0										
E	0										
A	0										
E	0										

Why is this zero?

What's the score from our BLOSSUM matrix for substituting H for P?

26

Next round!

		H	E	A	G	A	W	G	H	E	E
	0	0	0	0	0	0	0	0	0	0	0
P	0	0	0								
A	0	0	0								
W	0										
H	0										
E	0										
A	0										
E	0										

Terrible! Again, none of the possible give positive scores.  
We have to go a bit further in before we find a positive score...

27

A few more rounds, and a positive score at last!

		H	E	A	G	A	W	G	H	E	E
	0	0	0	0	0	0	0	0	0	0	0
P	0	0	0	0							
A	0	0	0	5							
W	0	0	0								
H	0										
E	0										
A	0										
E	0										

How did we get this one?

28

& a few more rounds...

		H	E	A	G	A	W	G	H	E	E
	0	0	0	0	0	0	0	0	0	0	0
P	0	0	0	0	0						
A	0	0	0	5	0						
W	0	0	0	0	2						
H	0	10	2	0	0						
E	0										
A	0										
E	0										

What does this mean?

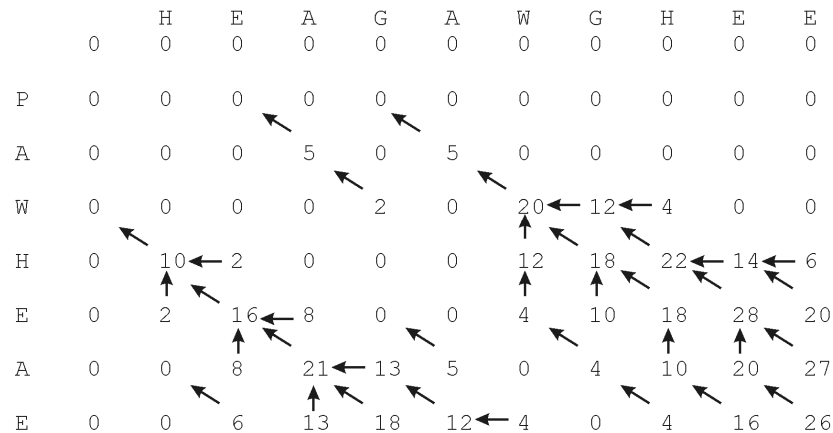
29

The whole thing filled in!

		H	E	A	G	A	W	G	H	E	E
	0	0	0	0	0	0	0	0	0	0	0
P	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	5	0	5	0	0	0	0	0
W	0	0	0	0	2	0	20	12	4	0	0
H	0	10	2	0	0	0	12	18	22	14	6
E	0	2	16	8	0	0	4	10	18	28	20
A	0	0	8	21	13	5	0	4	10	20	27
E	0	0	6	13	18	12	4	0	4	16	26

30

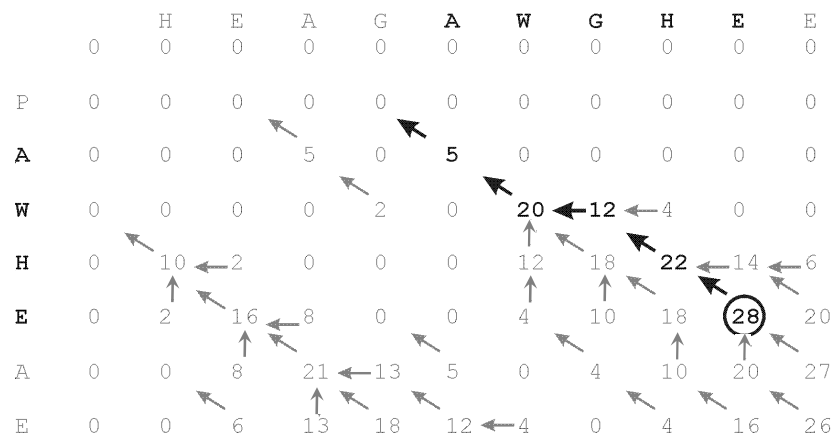
Now, find the optimal alignment using a **traceback** process:  
**Look for the highest score, then follow the arrows back.**  
**The alignment “grows” from right to left**



31

This gives the following alignment:     **AWGHE**  
   **AW-HE**

(Note: for gaps, the arrow points to the sequence that gets the gap)



32



**To align two sequences, we need to perform 3 steps:**

1. We need some way to decide which alignments are better than others.  
For this, we'll invent a way to give the alignments a "score" indicating their quality.
2. Align the two proteins so that they get the best possible score.
3. **Decide if the score is "good enough" for us to believe the alignment is biologically significant.**

33

This algorithm always gives the best alignment.

Every pair of sequences can be aligned in some fashion.

**So, when is a score "good enough"?**

How can we figure this out?

34

Here's one approach:

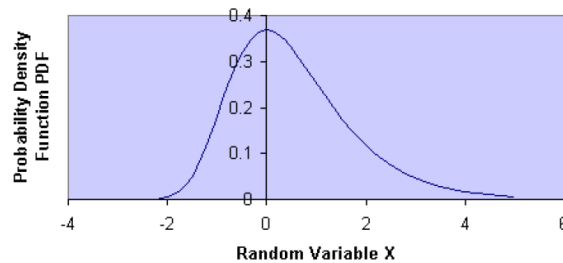
**Shuffle one sequence. Calculate the best alignment & its score.  
Repeat 1000 times.**

If we never see a score as high as the real one, we say the real score has <1 in a 1000 chance of happening just by luck.

But if we want something that only occurs < 1 in a million, we'd have to shuffle 1,000,000 times...

35

Luckily, alignment scores follow a well-behaved distribution, the **extreme value distribution**, so we can do a few trials & fit to this.



# random trials & their average score

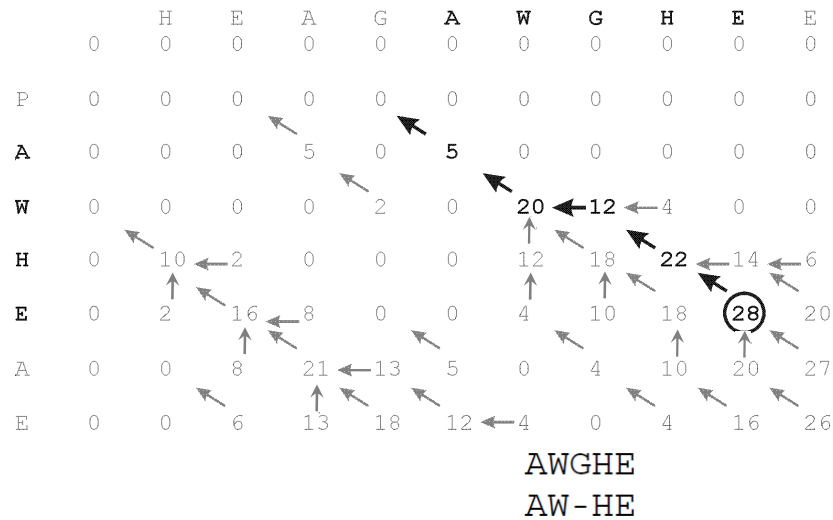
$$p(\text{max score} \leq X) \approx e^{-kNe^{\lambda(X-\mu)}}$$

This p-value gives the significance of your alignment.  
But, if we search a database and perform many alignments, we still need something more (next time).

Describe the shape & can be fit from a few trials

36

Some extensions: Local vs. global alignments  
How might you force the full sequences to align?



37

Some extensions: Local vs. global alignments  
How might you force the full sequences to align?

A few tiny changes:

Initialize only the top left cell of the path matrix to zero  
(not all top and left cells).

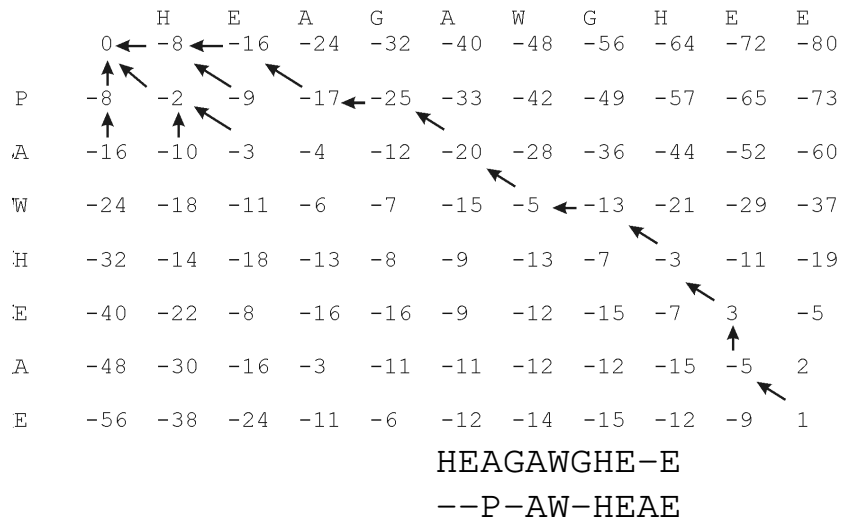
Leave the negative values (don't set them to zero).

The optimal alignment should start at the top left cell and  
finish at the bottom right cell of the path matrix.

Start the trace-back at the bottom right cell

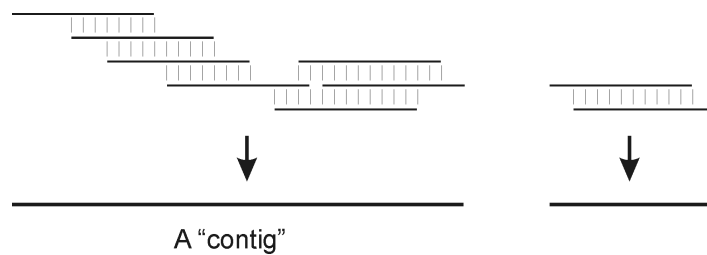
38

Some extensions: Local vs. global alignments  
 How might you force the full sequences to align?



39

Some extensions:  
 What about overlapping sequences?  
 e.g. as in 'shotgun sequencing' genomes where  
 'contigs' are built up from overlapping sequences



40

Some extensions:

What about overlapping sequences?

Modify global alignment to not penalize overhangs:

The optimal alignment should start at the top or left edge and finish at the bottom or right edge of the path matrix.

Set these boundary conditions :

$$F(i,0) = 0 \text{ for } i=1 \text{ to } n$$

$$F(0,j) = 0 \text{ for } j=1 \text{ to } m$$

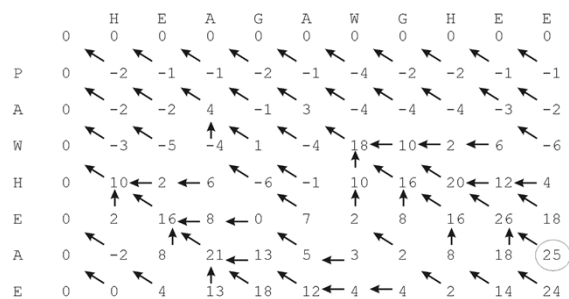
Start the traceback at the cell with the highest score on the right or bottom border

41

Some extensions:

What about overlapping sequences?

e.g. as in 'shotgun sequencing' genomes where 'contigs' are built up from overlapping sequences



(overhang = HEA)

GAWGHEE

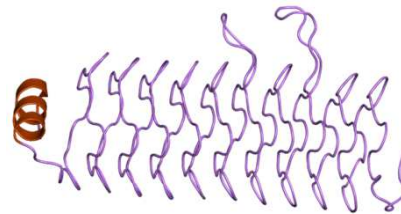
PAW-HEA

(overhang = E)

42

Some extensions:

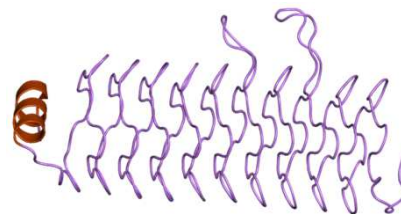
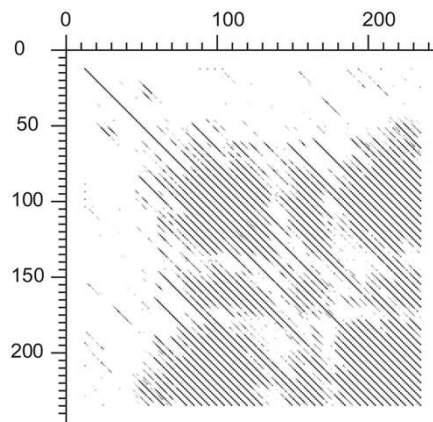
How might you find repetitive sequences?



Structure of the pentapeptide  
repeat protein HetL  
(from wiki, PMID18952182)

43

Align the sequence to itself and ignore the diagonal (optimal) alignment  
→ High-scoring off-diagonal alignments will be repeats



Structure of the pentapeptide  
repeat protein HetL  
(from wiki, PMID18952182)

Dot plot (quick visualization of  
sequence similarity)  
of the pentapeptide repeat  
protein HgIK protein vs. itself  
([http://en.wikipedia.org/wiki/Pentapeptide\\_repeat](http://en.wikipedia.org/wiki/Pentapeptide_repeat))

44