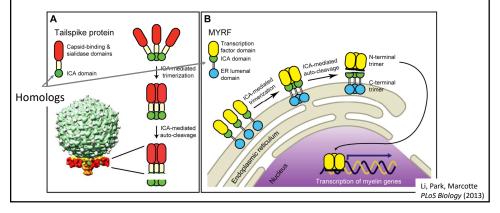
Typically, to be "biologically related" means to share a common ancestor. In biology, we call this homologous.

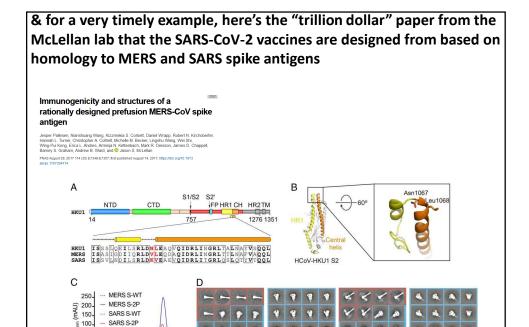
Two proteins sharing a common ancestor are said to be homologs. Homology often implies structural similarity & sometimes (not always) sequence similarity. A statistically significant sequence or structural similarity can be used to infer homology (common ancestry).

e.g., Myoglobin & Hemoglobin

In practice, searching for sequence or structural similarity is one of the most powerful computational approaches to discover a gene's function. We can often gain insight about a protein from its homologs.

For example, my lab discovered that myelinating the neurons in your brain reuses the same biochemical mechanism that phage use to make capsids. The key breakthrough was recognizing that the human and phage proteins contained homologous domains.





Elution volume (mL)

4 4 8 8

Sequence alignment algorithms such as BLAST, PSI-BLAST, FASTA, and the Needleman–Wunsch & Smith-Waterman algorithms arguably comprise some of the most important driver technologies of modern biology and underlie the sequencing revolution.

So, let's start learning bioinformatics algorithms by learning how to align two protein sequences.

### Live demo:

MVLSPADKTNVKAAWGKVGAHAGEYGAEALERMFLSFPTTKTYFPHFDLSHGSAQVKGHG KKVADALTNAVAHVDDMPNALSALSDLHAHKLRVDPVNFKLLSHCLLVTLAAHLPAEFTP AVHASLDKFLASVSTVLTSKYR

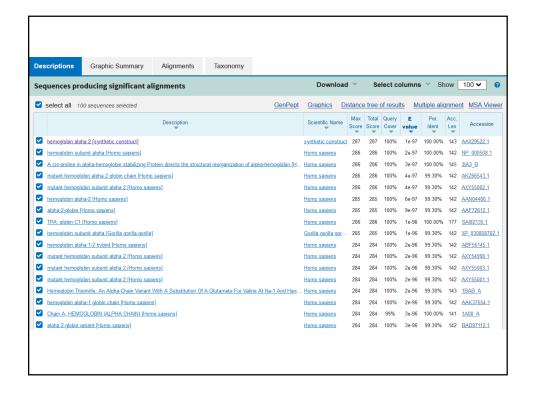
The next few slides show the data from searching this dbase (#'s may be a bit different from the live version):

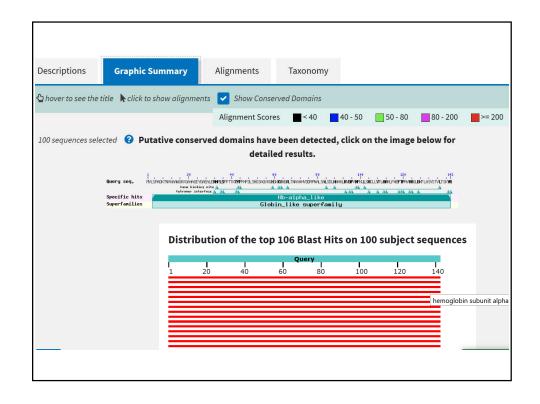
Title:All non-redundant GenBank CDS translations+PDB+SwissProt+PIR+PRF excluding environmental samples from WGS projects

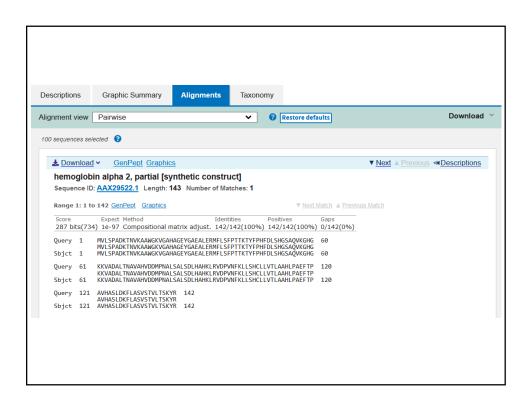
Molecule Type:Protein

Update date:2023/01/12

Number of sequences:522605238







# Protein sequence alignment

### Two biologically related proteins with similar sequences:

### Also biologically related (& fold up into the same 3D protein structure):

F1gA1 EAGNVKLKRGRLDTLPPRTVLDINQLVDAISLRDLSPDQPIQLTQFRQAWRVKAGQRVNVIASGD

A + P + L I+ R L P + I R+AW V+ G V V

F1gA3 LAALKQVTLIAGKHKPDAMATHAEELQGKIAKRTLLPGRYIPTAAIREAWLVEQGAAVQVFFIAG

### But these are biologically unrelated (& fold up into unrelated structures):

F1gA1 AGNVKLKRGRLDTLPPRTVLDINQLVDAISLRDLSPDQPIQLTQFRQA-WRVKAGQRVNVIASGD

AG+V K G + + PRT ++ I+ P PI +++A WRV A + V V+ GD

HvcPP AGHV--KNGTMRIVGPRTCSNVWNGTFPINATTTGPSIPIPAPNYKKALWRVSATEYVEVVRVGD

(FYI, we'll draw examples from Durbin et al., Biological Sequence Analysis, Ch. 1 & 2).

### To align two sequences, we need to perform 3 steps:

- 1. We need some way to decide which alignments are better than others
  - For this, we'll invent a way to give the alignments a "score" indicating their quality.
- 2. Align the two proteins so that they get the best possible score.
- 3. Decide if the score is "good enough" for us to believe the alignment is biologically significant.

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We'll treat mutations as independent events.

This allows us to create an *additive scoring scheme*. The score for a sequence alignment will be the sum of the scores for aligning each of the individual positions in two sequences.

What kind of mutations should we expect?

### Substitutions, insertions and deletions.

Insertions and deletions can be treated as equivalent events by considering one or the other sequence as the reference, and are usually called *gaps*.

substitution

gap

Let's consider two models:

First, a <u>random</u> model, where amino acids in the sequences occur independently at some given frequencies.

The probability of observing an alignment between x and y is just the product of the frequencies (q) with which we find each amino acid.

We can write this as:

What does the capital pi mean?

$$P(x, y \mid R) = \prod_{i}^{\nu} q_{x_i} \prod_{j} q_{y_j}$$

What's this mean?

What's this mean?

Second, a <u>match</u> model, where amino acids at a given position in the alignment arise from some common ancestor with a probability given by the joint probability  $p_{ab}$ .

So, under this model, the probability of the alignment is the product of the probabilities of seeing the individual amino acids aligned.

We can write that as:

What does the capital pi mean again?

$$P(x, y \mid M) = \prod_{i} p_{x_{i}y_{i}}$$

What's this mean?

What's this mean?

To decide which model better describes an alignment, we'll take the ratio:

$$\frac{P(x, y \mid M)}{P(x, y \mid R)} = \frac{\prod_{i} p_{x_{i}y_{i}}}{\prod_{i} q_{x_{i}} \prod_{j} q_{y_{j}}} = \prod_{i} \frac{p_{x_{i}y_{i}}}{q_{x_{i}} q_{y_{i}}}$$

What did these mean again?

Such a ratio of probabilities under 2 different models is called an *odds ratio*.

Where else have you heard odds ratios used?

Basically: if the ratio > 1, model *M* is more probable

if < 1, model *R* is more probable.

Now, to convert this to an <u>additive score</u> *S*, we can simply take the logarithm of the odds ratio (called the *log odds ratio*):

$$S = \sum_{i} s(x_i, y_i)$$

This is just the score for aligning one amino acid with another amino acid:

$$s(a,b) = \log\left(\frac{p_{ab}}{p_a p_b}\right)$$

Here written a and b rather than  $x_i$  and  $y_i$  to emphasize that this score reflects the <u>inherent preference</u> of the two amino acids (a and b) to be aligned.

Almost done with step 1...

### The last trick:

Take a big set of <u>pre-aligned</u> protein sequence alignments (that are correct!) and <u>measure</u> all of the pairwise amino acid substitution scores (the s(a,b)'s). Put them in a 20x20 *amino acid substitution matrix*:

```
A R N D C Q E G H I L K M F P S T W
 \begin{smallmatrix} Q & -1 & 1 & 0 & 0 & -3 & 7 & 2 & -2 & 1 & -3 & -2 & 2 & 0 & -4 & -1 & 0 & -1 & -1 & -1 & -3 \\ E & -1 & 0 & 0 & 2 & -3 & 2 & 6 & -3 & 0 & -4 & -3 & 1 & -2 & -3 & -1 & -1 & -1 & -3 & -2 & -3 \\ \end{smallmatrix} 
G 0 -3 0 -1 -3 -2 -3 8 -2 -4 -4 -2 -3 -4 -2 0 -2 -3 -3 -4
I -1 -4 -3 -4 -2 -3 -4 -4 -4 5 2 -3 2 0 -3 -3 -1 -3 -1
L -2 -3 -4 -4 -2 -2 -3 -4 -3 2 5 -3 3 1 -4 -3 -1 -2 -1
K -1 3 0 -1 -3 2 1 -2 0 -3 -3 6 -2 -4 -1 0 -1 -3 -2 -3
 \begin{smallmatrix} M & -1 & -2 & -2 & -4 & -2 & 0 & -2 & -3 & -1 & 2 & 3 & -2 & 7 & 0 & -3 & -2 & -1 & -1 & 0 & 1 \\ \end{smallmatrix} 
F -3 -3 -4 -5 -2 -4 -3 -4 -1 0 1 -4 0 8 -4 -3 -2 1
P -1 -3 -2 -1 -4 -1 -1 -2 -2 -3 -4 -1 -3 -4 10 -1 -1 -4 -3 -3
S 1 -1 1 0 -1 0 -1 0 -1 -3 -3 0 -2 -3 -1 5 2 -4 -2 -2
Y -2 -1 -2 -3 -3 -1 -2 -3 2 -1 -1 -2 0 4 -3 -2 -2 2
V 0 -3 -3 -4 -1 -3 -3 -4 -4 4 1 -3 1 -1 -3 -2 0 -3 -1 5
```

### This is the **BLOSUM50** matrix.

(The numbers are scaled & rounded off to the nearest integer):

What's the score for aspartate (D) aligning with itself? How about aspartate with phenylalanine (F)? Why?

```
 \hbox{A }\hbox{R }\hbox{N }\hbox{D }\hbox{C }\hbox{Q }\hbox{E }\hbox{G }\hbox{H }\hbox{I }\hbox{L }\hbox{K }\hbox{M }\hbox{F }\hbox{P }\hbox{S }\hbox{T }\hbox{W }\hbox{Y } 
N -1 -1 7 2 -2 0 0 0 1 -3 -4 0 -2 -4 -2 1 0 -4 -2 -3
Q -1 1 0 0 -3 7 2 -2 1 -3 -2 2 0 -4 -1 0 -1 -1 -1 -3
E -1 0 0 2 -3 2 6 -3 0 -4 -3 1 -2 -3 -1 -1 -1 -3 -2 -3
 \begin{smallmatrix} G & 0 & -3 & 0 & -1 & -3 & -2 & -3 & 8 & -2 & -4 & -4 & -2 & -3 & -4 & -2 & 0 & -2 & -3 & -3 & -4 \\ \end{smallmatrix} 
H \ -2 \quad 0 \quad 1 \ -1 \ -3 \quad 1 \quad 0 \ -2 \ 10 \ -4 \ -3 \quad 0 \ -1 \ -1 \ -2 \ -1 \ -2 \ -3 \quad 2 \ -4
I -1 -4 -3 -4 -2 -3 -4 -4 -4 5 2 -3 2 0 -3 -3 -1 -3 -1 4
L -2 -3 -4 -4 -2 -2 -3 -4 -3 2 5 -3 3 1 -4 -3 -1 -2 -1 1
K -1 3 0 -1 -3 2 1 -2 0 -3 -3 6 -2 -4 -1 0 -1 -3 -2 -3
 \begin{smallmatrix} M & -1 & -2 & -2 & -4 & -2 & 0 & -2 & -3 & -1 & 2 & 3 & -2 & 7 & 0 & -3 & -2 & -1 & -1 & 0 & 1 \\ \end{smallmatrix} 
F -3 -3 -4 -5 -2 -4 -3 -4 -1
                              0 1 -4 0 8 -4 -3 -2 1
 P \ -1 \ -3 \ -2 \ -1 \ -4 \ -1 \ -1 \ -2 \ -2 \ -3 \ -4 \ -1 \ -3 \ -4 \ 10 \ -1 \ -1 \ -4 \ -3 \ -3 
S 1 -1 1 0 -1 0 -1 0 -1 -3 -3 0 -2 -3 -1 5 2 -4 -2 -2
W -3 -3 -4 -5 -5 -1 -3 -3 -3 -3 -2 -3 -1
                                          1 -4 -4 -3 15
Y -2 -1 -2 -3 -3 -1 -2 -3 2 -1 -1 -2 0 4 -3 -2 -2 2 8 -1
```

# Using this matrix, we can score any alignment as the sum of scores of individual pairs of amino acids.

For example, the top alignment in our earlier example:

F1gA1 EAGNVKLKRGRLDTLPPRTVLDINQLVDAISLRDLSPDQPIQLTQFRQAWRVKAGQRVNVIASGD ++K+K+GRLDTLPP +L+ N A+SLR ++ QP+ R+ W +KAGQ V V+A G+ F1gA2 TLQDIKMKQGRLDTLPPGALLEPNFAQGAVSLRQINAGQPLTRNMLRRLWIIKAGQDVQVLALGE

gets the score:

$$S(FlgA1, FlgA2) = -1 - 2 - 2 + 2 + 4 + 6 + ... = 186$$

We also need to penalize **gaps**. For now, let's just use a constant penalty **d** for each amino acid gap in an alignment, *i. e.*:

the penalty for a gap of length g = -g\*d

### **PAM**



Margaret Dayhoff (1925-1983)
Developed point accepted
mutation matrices
(PAM matrices)

Calibrated for different evolutionary times
PAM-n = n substitutions per 100 residues
e.g. matrices from PAM1 to PAM250
measure PAM1,
calculate higher PAMs from that

<u>Explicit model of evolution</u>
(calculated using a phylogenetic tree)

### vs. BLOSUM



Steve and Jorja Henikoff Developed BLOSUM matrices

<u>Calibrated for different % identity sequences</u>
BLOSUM-*n* = for sequences of about *n* % identity averages substitution probabilities over sequence clusters, gives better estimates for highly divergent cases

<u>Implicit model of evolution</u>
(calculated from blocks of aligned sequences)

### To align two sequences, we need to perform 3 steps:

1. We need some way to decide which alignments are better than others.

For this, we'll invent a way to give the alignments a "score" indicating their quality.

- 2. Align the two proteins so that they get the best possible score.
- 3. Decide if the score is "good enough" for us to believe the alignment is biologically significant.

### A sense of scale:

There are  $\binom{2n}{n}pprox rac{2^{2n}}{\sqrt{\pi n}}$  possible global alignments between two sequences of length n if we use gaps

So, with 2 sequences of length 100, that's  $> 10^{60}$  possible alignments

We'll use something called dynamic programming.

This is **mathematically guaranteed** to find the best scoring alignment, and uses *recursion*. This means problems are broken into subproblems, which are in turn broken into sub-problems, etc, until the simplest sub-problems can be solved.

We're going to find the best *local* alignment—the best matching internal alignment—without forcing <u>all</u> of the amino acids to align (i.e. to match *globally*).

i.e., this 
$$\longrightarrow$$
  $\begin{tabular}{lll} ATGCAT \\ ATGCAT \\ \\ Not this &\longrightarrow & ACGTTATGCATGACGTA \\ & -C---ATGCAT----T- \\ \\ \end{tabular}$ 

### Here's the main idea:

We'll make a *path matrix*, showing the possible alignments and their scores. There are simple rules for how to fill in the matrix. *This will test all possible alignments & give us the top-scoring alignment between the two sequences.* 

### Here are the rules:

For a given square in the matrix F(i,j), we look at the squares to its left F(i-1,j), top F(i,j-1), and top-left F(i-1,j-1). Each should have a score.

We consider **3 possible events** & **choose the one scoring the highest**:

(1) 
$$x_i$$
 is aligned to  $y_i$ 

$$F(i-1,j-1) + s(x_i,y_i)$$

(2) 
$$x_i$$
 is aligned to a gap

(3) 
$$y_i$$
 is aligned to a gap

$$F(i,j-1)-d$$

For this example, we'll use d = 8. We also set the left-most & top-most entries to zero.

### Just two more rules:

If the score is negative, set it equal to zero.

At each step, we also keep track of which event was chosen by drawing an arrow from the cell we just filled back to the cell which contributed its score to this one.

That's it! Just repeat this to fill the entire matrix.

Here we go! Start with the borders & the first entry.

H E A G A W G H E E
O O O O O O O O O O O O O

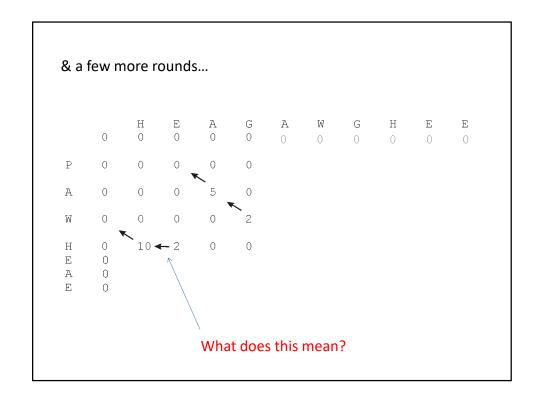
P O O
A O
W O
H O
E O
A O
E O

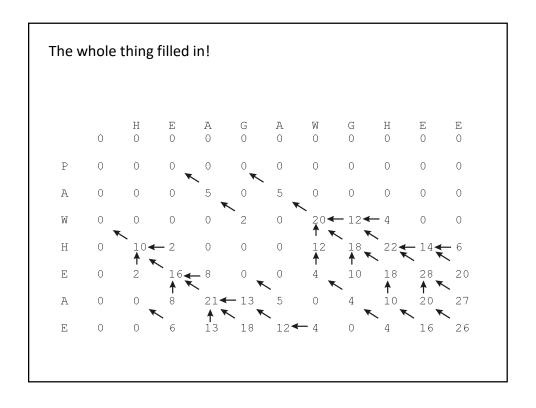
Why is this zero?
What's the score from our BLOSSUM matrix for substituting H for P?

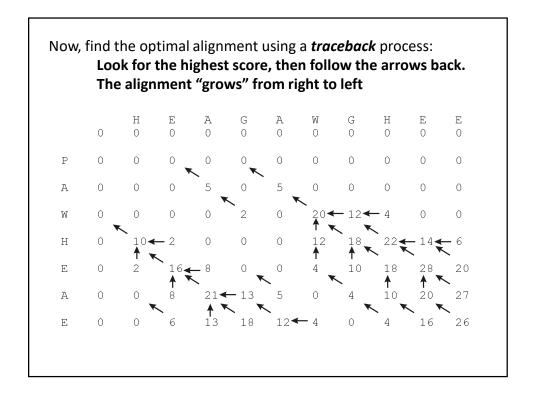
### Next round! Α G A W G Н Ε Ε Η 0 0 Α 0 0 0 W Η Ε Α 0

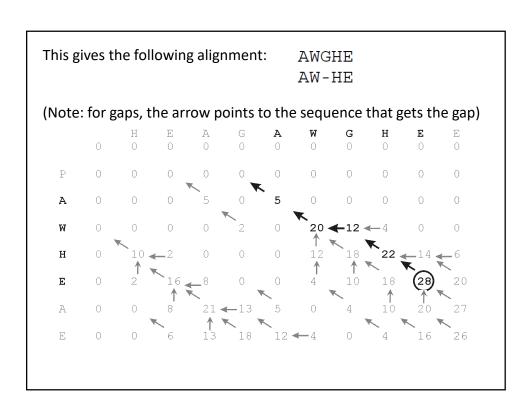
Terrible! Again, none of the possible give positive scores. We have to go a bit further in before we find a positive score...

A few more rounds, and a positive score at last! Η G Α W G Н E E 0 Α 0 0 Н 0 Ε 0 A 0 How did we get this one?









### To align two sequences, we need to perform 3 steps:

1. We need some way to decide which alignments are better than others.

For this, we'll invent a way to give the alignments a "score" indicating their quality.

- 2. Align the two proteins so that they get the best possible score.
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This algorithm <u>always</u> gives the best alignment.

Every pair of sequences can be aligned in some fashion.

So, when is a score "good enough"?

How can we figure this out?

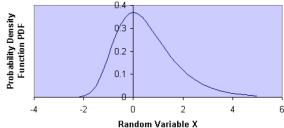
Here's one approach:

Shuffle one sequence. Calculate the best alignment & its score. Repeat 1000 times.

If we never see a score as high as the real one, we say the real score has <1 in a 1000 chance of happening just by luck.

But if we want something that only occurs < 1 in a million, we'd have to shuffle 1,000,000 times...

Luckily, alignment scores follow a well-behaved distribution, the *extreme value distribution*, so we can do a few trials & fit to this.

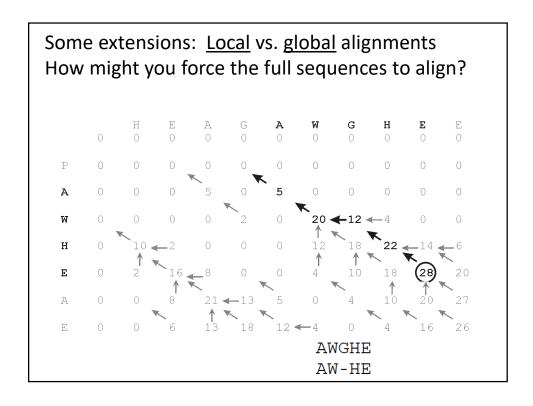


# random trials & their average score

$$p(\max \text{score} \leq X) \approx e^{-kNe^{\lambda(X-\mu)}}$$

This p-value gives the significance of your alignment. But, if we search a database and perform many alignments, we still need something more (next time).

Describe the shape & can be fit from a few trials



# Some extensions: <u>Local</u> vs. <u>global</u> alignments How might you force the full sequences to align?

A few tiny changes:

Initialize only the top left cell of the path matrix to zero (not all top and left cells).

Leave the negative values (don't set them to zero).

The optimal alignment should start at the <u>top left</u> cell and finish at the <u>bottom right</u> cell of the path matrix.

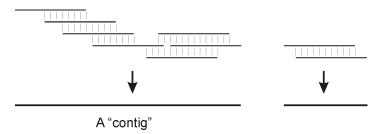
Start the trace-back at the bottom right cell

# Some extensions: <u>Local</u> vs. <u>global</u> alignments How might you force the full sequences to align?

### Some extensions:

What about overlapping sequences?

e.g. as in 'shotgun sequencing' genomes where 'contigs' are built up from overlapping sequences



### Some extensions:

## What about overlapping sequences?

Modify global alignment to not penalize overhangs:

The optimal alignment should start at the <u>top</u> or <u>left</u> edge and finish at the <u>bottom</u> or <u>right</u> edge of the path matrix.

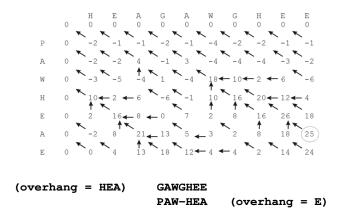
Set these boundary conditions:

$$F(i,0) = 0$$
 for  $i=1$  to  $n$   
 $F(0,j) = 0$  for  $j=1$  to  $m$ 

Start the traceback at the cell with the highest score on the <u>right</u> or <u>bottom</u> border

### Some extensions:

What about overlapping sequences?
e.g. as in 'shotgun sequencing' genomes where
'contigs' are built up from overlapping sequences



# Some extensions: How might you find repetitive sequences? Structure of the pentapeptide repeat protein HetL (from wiki, PMID18952182)

